semi-covariants R, S_i, $T_{l\tau}$ may be expressed as semi-covariants of (A) simply by replacing $y_i^{(\tau)}$ by $t_{i\tau}$, π_{ikl} by u_{ikl}, and π_{ikl} by v_{ikl}, where u_{ikl} and v_{ikl} are functions of the coefficients of (A) and their derivatives which appear in the expressions for π_{ikl} and π'_{ikl}.

If the transformation (1) and the corresponding transformations for the derivatives of y_i are made infinitesimal, and the resulting system of partial differential equations for the semi-covariants is set up, it is found that there are exactly mn relative semi-covariants which are not seminvariants. We thus have the proper number of semi-covariants, but it remains to show that they are independent.

A comparison of R and S_i with the corresponding semi-covariants[3] for the special case of (A) where $m = 2$ shows R and S_i to be independent. Again, the functional determinant of $T_{l\tau}$ with respect to $y_i^{(\tau)}$ $(i=1,2,\ldots,n)$ for each value of $\tau = 1, 2,\ldots\ldots, m-1$ shows[3] that $T_{l\tau}$ are independent, among themselves and of R, of S_i and of the seminvariants.

We have now proved the following theorem:

All semi-covariants are functions of seminvariants and of R, S_i $(i = 1$ $2,\ldots\ldots\ldots, n-1)$, $T_{l\tau}$ $(l = 0, 1,\ldots\ldots\ldots, n-1; \tau = 1, 2,\ldots\ldots\ldots m-1)$.

[1] Wilczynski, E. J., *Projective Differential Geometry of Curves and Ruled Surfaces*, Teubner, Leipzig, Chap. I.

[2] Stouffer, these PROCEEDINGS, **6**, 1920 (645–8).

[3] Stouffer, *London, Proc. Math. Soc.*, (Ser. 2), **17**, 1919 (337–52).

AN ALGORISM FOR DIFFERENTIAL INVARIANT THEORY

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1. Comprehensive as the existent theory of differential parameters is, as related to quantics

$$F = (a_0, a_1, \ldots, a_m)\ (dx_1, dx_2)^m\ (aj = aj(x_1, x_2)),$$

under arbitrary functional transformations

$$(1) \qquad x_i = x_i\,(y_1, y_2)\ (i = 1, 2),$$

developments of novelty relating to the foundations result when emphasis is placed upon the domains within which concomitants of such classes may be reducible, particularly a certain domain $R(\iota,T,\Delta)$ defined in part by certain irrational expressions in the derivatives of the arbitrary functions occurring in the transformations. For a given set of forms F all differential parameters previously known are functions in R of certain elementary invariants, which we designate as invariant elements, and their derivatives. The theory of invariant elements serves, therefore, to unify known theories and, for the various categories of parameters, gives a means of classification.

It leads also to systems of certain new types which I designate as orthogonal[1] and the extended orthogonal types of differential parameters, and methods of enumeration relating to these systems, both general and particular.

In the paper of which the present article is an abstract, references to the literature are only incidental to the developments but mention may be given to memoirs by Christoffel,[2] Ricci and Levi-Civita,[3] to the tract of J. E. Wright,[4] containing bibliography, and to the symbolical theory of Maschke.[5]

2. The poles of the transformations on the differentials,

$$T : dx_i = \frac{\partial x_i}{\partial y_1} dy_1 + \frac{\partial x_i}{\partial y_2} dy_2 \ (i = 1, 2),$$

are the roots of the linear forms

$$h_{\pm 1} = 2\frac{\partial x_2}{\partial y_1} dx_1 + \left(\frac{\partial x_2}{\partial y_2} - \frac{\partial x_1}{\partial y_1} \pm \Delta\right) dx_2, \tag{2}$$

where

$$\Delta = \left[\left(\frac{\partial x_2}{\partial y_2} - \frac{\partial x_1}{\partial y_1}\right)^2 + 4\frac{\partial x_1}{\partial y_2}\frac{\partial x_2}{\partial y_1}\right]^{\frac{1}{2}},$$

and we may place $h_{\pm 1}$ equal to exact differentials

$$\mathbf{T} : df_{+1} = h_{+1},\ df_{-1} = h_{-1},$$

thus obtaining another transformation upon the differentials of the same degree of arbitrariness as T, since the functional determinant of f_{+1}, f_{-1} will not vanish when $\Delta \neq 0$. The quantics $df_{\pm 1}$ are relative differential covariants appertaining to a domain $R(1, T, \Delta)$ whose defining quantities are the y_1, y_2 derivatives of x_1 and x_2, and the expression Δ. The covariant relations are

$$df'_{+1} = \rho_{+1}^{-1} df_{+1},\ df_{-1} = \rho_{-1}^{-1} df_{-1}, \tag{3}$$

where

$$\rho_{\pm 1} = \frac{1}{2}\left(\frac{\partial x_1}{\partial y_1} + \frac{\partial x_2}{\partial y_2} \pm \Delta\right),$$

are the factors in $R(1, T, \Delta)$ of the determinant D of T.

3. Let

$$\partial x_1/\partial y_1 = \alpha_1, \partial x_1/\partial y_2 = \alpha_2, \partial x_2\partial/y_1 = \beta_0, \partial x_2/\partial y_2 = \beta_1,$$
$$\partial f/\partial x_i = f_i \ (i = 1, 2),$$

and write, after Maschke, the quantic F as the m-th power of a symbolical exact differential,

$$F = (df)^m = (f_1 dx_1 + f_2 dx_2)^m = (d\varphi)^m.$$

Then when F is transformed by the inverse of $\mathbf{T}$, and the result expanded, the coefficients φ_{m-2i} $(i = o, \ldots\ldots, m)$ of the terms in df_{+1}, df_{-1} are differential parameters of the domain $R(1, T, \Delta)$, forming, with $df_{\pm 1}$, a complete system in this domain. Their explicit form is

$\varphi_{m-2i}=[(\gamma_1-\Delta)f_1-2\beta_0 f_2]^{m-i}[-(\gamma_1+\Delta)f_1+2\beta_0 f_2]^i(-4\beta_0\Delta)]^{-m}(i=o,..,m)$
and the invariant relations are

$$\varphi'_{m-2i} = \rho_{+1}^{m-2i}D^i\varphi_{m-2i}\ (i=o,\ \ldots,\ m). \tag{4}$$

The functions φ_{m-2i}, df_{+1}, df_{-1} are the invariant elements.

4. By differentiation of the equations (4) relations are obtained as follows:

$$\frac{\partial^r\varphi'_{m-2i}}{\partial y_1^{r-s}\partial y_2^s} = \alpha_1^{r-s}\beta_1^s\rho_{+1}^{m-2i}D^i\frac{\partial^r\varphi_{m-2i}}{\partial x_1^{r-s}\partial x_2^s}+\delta_r^{(i)},$$

$$(r = o,\ 1,\ \ldots\ldots,;\ s = o,\ \ldots\ldots,\ r;\ i = o,\ \ldots\ldots,\ m).$$

As a result, employing the transvectant symbol[5]

$$U_1V_2 - U_2V_1 = (U,\ V)$$

we obtain a general category of relative differential parameters,

$$\psi_{m-2i}^{(r)} = (\ldots((\varphi_{m-2i},\ Q^{(i)}),\ Q^{(i)}),\ldots,Q^{(i)})\quad (Q^{(i)}=\rho_{+1}^{m-2i}D^i),$$

the number of iterations being r, and

$$\psi_{m-2i}^{(r)\prime} = \rho_{+1}^{m-2i}D^{i+r}\psi_{m-2i}^{(r)};\ (i=o,\ldots,m). \tag{5}$$

We define $\psi_{m-2i}^{(o)}$ to be φ_{m-2i}.

Write

$$P_{+1} = \prod_{k=o}^{r}\psi_m^{(k)Xok}\ \psi_{m-2}^{(k)X_1k}\ldots\psi_{-m}^{(k)Xmk}df_{+1}^{\sigma_2}df_{-1}^{\sigma_1}$$

and let the conjugate product be P_{-1};

$$P_{-1} = \prod_{k=o}^{r}\psi_{-m}^{(k)Xok}\psi_{-m+2}^{(k)X_1k}\ldots\psi_m^{(k)Xmk}df_{+1}^{\sigma_2}df_{-1}^{\sigma_1},$$

then the parameters of the extended orthogonal type are defined to be those which can be generated in totality by forming such rational expressions in $\psi_{m-2i}^{(k)}$, $df_{\pm1}$ as simplify by multiplication into functions appertaining to the domain $R(\mathit{1},T,0)$, free, that is, from the expression Δ. The essential forms from which to construct this totality are evidently $P_{+1} \pm P_{-1}$. When $r = 0$ the type is called orthogonal.[1]

Finite complete systems can be derived in this theory. In fact the products P form a Hilbert system of monomials whence it follows that a complete system of concomitants of the extended orthogonal type is furnished by the finite set of irreducible solutions of the diophantine equation

$$\sum_{k=o}^{r}\sum_{l=o}^{m}(m-2l)X_{kl}-\sigma_1+\sigma_2=o. \tag{6}$$

Particular systems of the orthogonal type have been constructed by the present writer in this and previous papers for the quantics of orders one to six inclusive, the system of the sextic for example being composed of 31 parameters. For the case $r = 1$, $m = 2$ the system comprises eleven concomitants of the extended type as follows:

$$\varphi_0,\ \psi_0^{(1)},\ n = \varphi_2\varphi_{-2},\ o = (\varphi_2,\ \rho_{+1}^2)\ (\varphi_{-2},\ \rho_{-1}^2),$$
$$p_{\pm1} = \varphi_2(\varphi_{-2},\ \rho_{-1}^2)\ \pm\ \varphi_{-2}(\varphi_2,\ \rho^2{}_{+1}),$$

$$q_{\pm 1} = (\varphi_2, \rho^2_{+1})df^2_{+1} \pm (\varphi_{-2}, \rho^2_{-1})df^2_{-1},$$
$$r_{\pm 1} = \varphi_2 \, df^2_{+1} \pm \varphi_{-2} df^2_{-1}, \; s = df_{+1} \, df_{-1}.$$

5. A paper giving the above and other developments in particular is to appear in the September (1921) number of the *Annals of Mathematics*. An article relating to the rôle of invariant elements[6] in algebraic orthogonal and boolean invariant theory appeared in the *Transactions* of the American Mathematical Society, vol. 20 (1919), and a paper containing like theory for invariants of relativity and modular invariants appeared in the same *Transactions* for the year 1920. Further researches are in progress.

[1] Elliott, *Proc. London Math. Soc.*, **33** (1901).
[2] Christoffel, *Crelle, J. Math.*, **70** (1869).
[3] Ricci and Levi-Civita, *Math. Ann.*, **54** (1901).
[4] J. E. Wright, *Invariants of Quadratic Differential Forms*, Cambridge tracts (1908).
[5] Maschke, *Trans. Amer. Math. Soc.*, **1** (1900); **4** (1903).
[6] O. E. Glenn, *Ann. Math.*, **20** (1918).

THE EFFECT OF TEMPERATURE AND OF THE CONCENTRATION OF HYDROGEN IONS UPON THE RATE OF DESTRUCTION OF ANTISCORBUTIC VITAMIN (VITAMIN C)

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In these experiments the vitamin was determined by means of feeding experiments with guineapigs according to the general method made familiar by Hess' recent monograph.[1] Cohen and Mendel,[2] Givens and McCluggage,[3] and the workers at the Lister Institute[4] among other investigators of the antiscorbutic vitamin, have emphasized the importance of basal diets which shall supply all other nutritive requirements and yet furnish none of the vitamin in question or only negligible traces of it. Further investigation of this point led the present writers to adopt the following as an improvement upon the diets previously proposed.

Basal Ration Used.—Ground whole oats were mixed with skimmed milk powder which had been heated sufficiently to ensure destruction of such antiscorbutic vitamin as it might contain without so changing the flavor as to cause it to be refused by the experimental animals. The heat treatment necessary to ensure complete destruction of vitamin C in the skimmed milk powder should be determined by each investigator for his own material and technique of heating. In our experiments two hours' heating